

EFFECT OF CONFINEMENT ON CURVATURE DUCTILITY OF REINFORCED CONCRETE BEAMS

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A THESIS SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
In
Structural Engineering
By

NAREN CHAKRAVARTHY KAMINENI
(Roll No. 212CE2033)



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UNDER GUIDENCE OF

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CERTIFICATE

This is to certify that the Thesis Report entitled “EFFECT OF CONFINEMENT ON CURVATURE DUCTILITY OF REINFORCED CONCRETE BEAMS”, submitted by Mr. NAREN CHAKRAVATHY KAMINENI bearing Roll no. 212CE2033 in partial fulfillment of the requirements for the award of Master of Technology in Civil Engineering with specialization in “Structural Engineering” during session 2013-2014 at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

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ABSTRACT

It is a fact that the strength and ductility of the concrete is highly dependent on the confinement level provided by the lateral reinforcement. In the current design codes design of strength is separated with deformability. Evaluation of deformability is independent of some key parameters of concrete and steel.

In the present study curvature ductility of a RCC beams with different level of confinements are calculated analytically following Hong K N and Han S H (2005) Model and Saatcioglu and Razvi (1992) Model and compared with experimental results.

Six rectangular RCC beams having same cross section and main reinforcements are analysed by using OPENSEES software. Different level of lateral confinement in beams is induced by two legged and three legged stirrups provided with three different spacing. For experimental study six RCC beams are cast with stirrups provided at spacing of 100 mm, 150 mm and 250 mm. Three beams are cast with two legged and three beams are cast with three legged stirrups.

Analytical observation is that the curvature ductility increases with decrease in spacing of stirrups and increase in number of legs of stirrups i.e. lateral confinement increases the curvature ductility of beam . The variation with respect to spacing is more compared to number of legs of stirrups. It is proven by using both models.

The same trends are observed through experimental results.

Analytical results following Saatcioglu and Razvi (1995) Model are found to be in well agreement with the experimental results.

ACKNOWLEDGEMENT

The satisfaction and euphoria on the successful completion of any task would be incomplete without the mention of the people who made it possible whose constant guidance and encouragement crowned out effort with success.

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This acknowledgement would not be complete without expressing my sincere gratitude to my parents, brother and brother-in-law for their love, patience, encouragement, and understanding. My Parents are the source of my motivation and inspiration throughout my work. Finally I would like to dedicate my work and this thesis to my parents.

Naren Chakravarthy Kamineni.

TABLE OF CONTENT

	Page
ABSTRACT.....	(i)
ACKNOWLEDGEMENTS.....	(ii)
LIST OF FIGURES.....	(v)
LIST OF GRAPHS.....	(vi)
LIST OF TABLES.....	(vii)
CHAPTER 1 INTRODUCTION.....	01
1.1 Organization of thesis.....	02
CHAPTER 2 LITERATURE REVIEW.....	04
2.1 Stress-Strain curve as per IS 456:200.....	03
2.2 Hong K N and Han S H (2005) model for confined concrete.....	04
2.3 Saatcioglu and Razvi (1992) model for confined concrete.....	05
2.4 Modified Mander's and Fardis et al.....	06
CHAPTER 3 THEORY AND FORMULATIONS.....	08
3.1 Theory.....	09
3.2 Introduction to <i>OPENSEES</i>	10
3.3 Analysis of various confinement models.....	12
3.4 Hong and Han Model (2005).....	14
3.5 Saatcioglu and Razvi Model (1992).....	15-16
3.6 Mander's Model (2001).....	17
CHAPTER 4 ANALYTICAL RESULTS AND DISCUSSIONS.....	18
4.1 Moment vs. Curvature as per Hong K N and Han S H Model.....	19-20
4.2 Moment vs. Curvature as per Saatcioglu and Razvi Model.....	21-23
4.3 Comparison of Results.....	24-25

4.4 Discussions.....	26
CHAPTER 5 EXPERIMENTAL SET UP.....	27
5.1 Material Properties.....	28
5.1.1 Concrete.....	28
5.1.2 Reinforcing Steel.....	29
5.2 Casting of Beams.....	29
5.3 Tests and Results.....	30
5.3.1 Moment vs. Curvature for Beam 1.....	31
5.3.2 Moment vs. Curvature for Beam 2.....	32
5.3.3 Moment vs. Curvature for Beam 3.....	33-34
5.3.4 Moment vs. Curvature for Beam 4.....	35-36
5.3.5 Moment vs. Curvature for Beam 5.....	37
5.3.6 Moment vs. Curvature for Beam 6.....	38-39
5.4 Comparison of Results.....	40-42
CHAPTER 6	
CONCLUSION.....	43
CHAPTER 7	
REFERENCES.....	45

LIST OF FIGURES:-

S.NO	FIGURE	PAGE NO.
3.1	Stress Strain curve in <i>Opensees</i>	11
5.1	Testing and Crack pattern in Beam 1	28
5.2	Crack pattern in Beam 2	29
5.3	Failure in Beam 3	30
5.4	Crack pattern in Beam 4	32
5.5	Crack pattern in Beam 5	35
5.6	Testing of Beam 6	36
5.7	Crack pattern in Beam 6.	37

S.NO	LIST OF GRAPHS	PAGE NO
2.1	Stress-Strain as per IS 456:2000	06
2.2	Hong K N and Han S H (2005) Model	07
2.3	Saatcioglu and Razvi (1992) Model	09
4.1	Moment vs. Curvature for Beam 1 as per Hong K N and Han S H	20
4.2	Moment vs. Curvature for Beam 2 as per Hong K N and Han S H	20
4.3	Moment vs. Curvature for Beam 3 as per Hong K N and Han S H	22
4.4	Moment vs. Curvature for Beam 4 as per Hong K N and Han S H	23
4.5	Moment vs. Curvature for Beam 5 as per Hong K N and Han S H	25
4.6	Moment vs. Curvature for Beam 6 as per Hong K N and Han S H	25
4.7	Moment vs. Curvature for Beam 1 as per Saatcioglu and Razvi Model	26
4.8	Moment vs. Curvature for Beam 2 as per Saatcioglu and Razvi Model	28
4.9	Moment vs. Curvature for Beam 3 as per Saatcioglu and Razvi Model	29
4.10	Moment vs. Curvature for Beam 4 as per Saatcioglu and Razvi Model	31
4.11	Moment vs. Curvature for Beam 5 as per Saatcioglu and Razvi Model	32
4.12	Moment vs. Curvature for Beam 6 as per Saatcioglu and Razvi Model	33
5.1	Comparison of Experimental Results (2-legged and 3-legged)	34
5.2	Comparison of Experimental Moment Vs. Curvature with the Hong K N Han S H Model (2-legged)	34
5.3	Comparison of Experimental Vs. the Hong K N Han S H Model (3-legged)	35
5.4	Comparison of Experimental Vs. the Saatcioglu and Razvi Model (2-legged)	35
5.5	Comparison of Experimental Vs. the Saatcioglu and Razvi Model (3-legged)	36

LIST OF TABLES:-

S.NO	TABLE	PAGE NO
4.1	Comparison of models	28
5.1	Design mix proportions	29
5.2	Concrete strength at 28 days	29
5.3	Tensile strength of reinforcing steel bars	30
5.4	Moment Vs. Curvature for Beam 1	31
5.5	Moment Vs. Curvature for Beam 2	32
5.6	Moment Vs. Curvature for Beam 3	33
5.7	Moment Vs. Curvature for Beam 4	34
5.8	Moment Vs. Curvature for Beam 5	35
5.9	Moment Vs. Curvature for Beam 6	36

Chapter-1

INTRODUCTION

1. INTRODUCTION

It is well known that the strength and ductility of concrete are highly dependent on the level of confinement provided by level of the lateral reinforcement. In the flexural design of reinforced concrete (RC) beams, the strength and deformability, which are interrelated, need to be considered simultaneously. However, in current design codes, design of strength is separated with deformability, and evaluation of deformability is independent of some key parameters, like concrete strength, steel yield strength and confinement content. Hence, provisions in current design codes may not provide sufficient deformability for beams. In this thesis a detailed study is presented on ductility behavior of RC beams with confinement by experimentally and analytically. To investigate the influence of the transverse reinforcing ratio on the beam ductility, an experimental program is conducted. Six no's of beams are cast with varying c/c spacing between stirrups of two legged and three legged.

In the seismic design of reinforced concrete beams of structures, the potential plastic hinge regions need to be carefully detailed for ductility in order to ensure that the shaking from large earthquakes will not cause collapse. Adequate ductility of members of reinforced concrete frames is also necessary to ensure that moment redistribution can occur. Previous tests have shown that the confinement of concrete by suitable arrangements of transverse reinforcement results in a significant increase in both the strength and the ductility of the member. In particular, the strength enhancement from confinement and the slope of the descending branch of the concrete stress-strain curve have a considerable influence on the flexural strength and ductility of reinforced concrete beams.

The cover concrete will be unconfined and will eventually become ineffective after maximum allowed strain is attained, but the core concrete will continue to carry stress at high strains. The compressive stress distributions for the core and cover concrete are defined by confined and unconfined concrete stress-strain relations. Good confinement of the core concrete is essential if the beam is to have ductility. The deformability of RC flexural members depends upon a number of factors, including percentage of tensile reinforcement, percentage of compressive reinforcement, percentage of lateral reinforcement and strength of concrete. Investigation regarding ductility of flexural members utilizing normal weight aggregate and light weight aggregate has been explored in number of studies. Although adequate flexural ductility is essential for structures in high seismicity regions, many serious problems relating to the behavior of RC structures under severe seismic action can be traced due to the poor detailing of reinforced concrete.

Knowledge of post peak deformation characteristics of reinforced concrete members are very desirable for proper understanding of the contribution of lateral reinforcement and to understand the failure mechanisms under seismic conditions where, higher ductility demands are placed on reinforced concrete members.

1.1. OBJECTIVE:

The objective of the present work is to study the effect of different level of confinements on curvature ductility of an RCC beams. The analytical study is done following Hong K N and Han S H (2005) Model and Saatcioglu and Razvi (1992) Model .The study is further followed by experimental investigation.

Six rectangular RCC beams having same cross section and main reinforcements are analyzed by using OPENSEES software. Different level of lateral confinement in beams is induced by two legged and three legged stirrups provided with three different spacing.

The experimental investigation consists of six RCC beams cast with stirrups provided at spacing of 100 mm, 150 mm and 250 mm. Three beams are cast with two legged and three beams are cast with three legged stirrups.

The analytical results are compared with experimental results.

Chapter-2

LITERATURE REVIEW

2. LITERATURE REVIEW

A number of studies have generated very useful information on the strength and deformation characteristics of reinforced concrete members. However these studies are limited to ultimate load stage and failure modes, and there is no information available on post peak stage deformation of reinforced concrete members. It has been pointed by number of investigators that the testing methodology influences the mode of failure and post peak behavior of concrete. For example the failure of concrete under uncontrolled compressive loading cause brittle type failure where as under controlled condition relatively ductile failure occurs. It would be too expensive to design a structure based on the “elastic” spectrum, and the code (IS 1893) allows the use of a “Response Reduction Factor” (R), to reduce the seismic loads. But this reduction will be possible, if sufficient ductility is in-built through proper design of the structural elements. Hence to get a correct response *non-linear analysis* of RCC structures should be carried out. The inelastic analysis exhibits behaviour beyond the yielding stage which can be represented in terms of formation of plastic hinges, redistribution of moments etc.

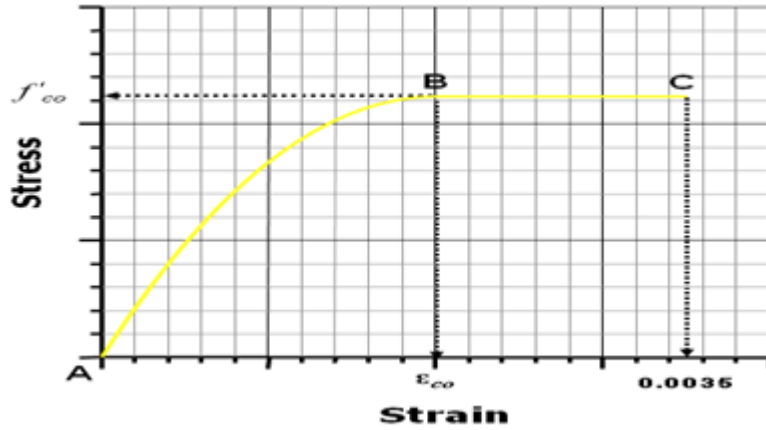
Ductility in a structure can be achieved by formation of plastic hinges at appropriate locations in the structural frame. The ductility of plastic hinge can be determined from the shape of the moment curvature relations. Moment curvature relation for an RCC beam can be determined if stress-strain relations for concrete and steel are known.

The ductility of RCC member can be drastically increased by suitable arrangement of stirrups causing confinement of core concrete. Hence during design stress-strain curve for confined concrete must be considered. Several models are available for stress-strain relation of confined concrete.

2.1) Stress-Strain Curves for concrete:

As per IS-456:2000:

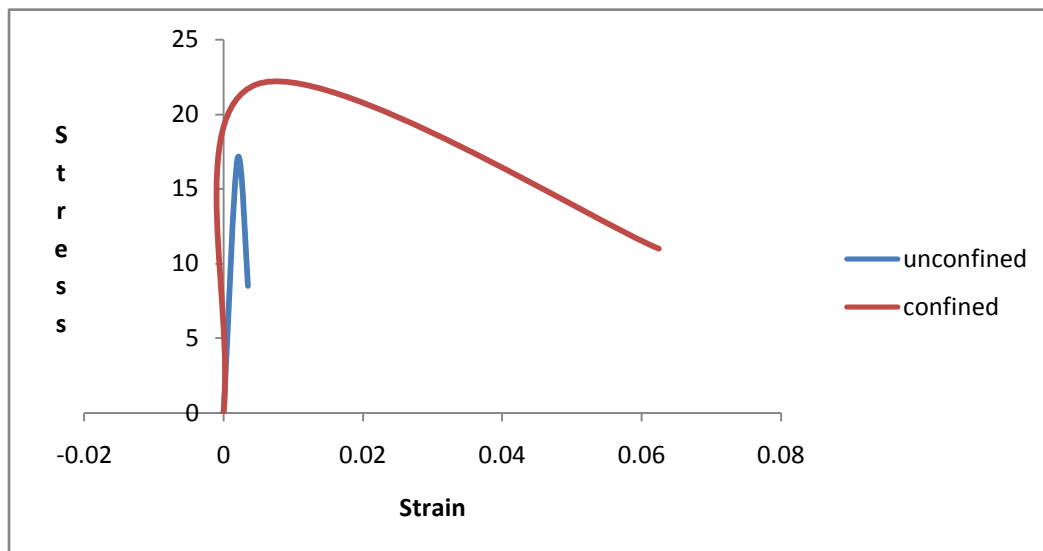
- a) Descending branch in the post-peak region not accounted for.
- b) Enhancement/reduction in ductility (and/or strength) due to confinement, grades of concrete and steel, bond, shear, etc., not accounted for.



Graph 2.1. Stress-Strain as per IS 456:2000

2.2) HONG K N and Han S H (2005) Model:

This model proposed two equations for ascending and descending branches of the stress-strain curve by considering the properties of the lateral reinforcement such as diameter, spacing, yield strength, configuration and longitudinal reinforcement. A graph is shown here which will differentiate between confined and unconfined concrete.



Graph 2.2. Stress-Strain Curve

Equations For ascending and descending branches of confined concrete:

Ascending branch:

$$f_c = f_{cc} \left\{ 1 - \left[1 - \frac{\epsilon_c}{\epsilon_{cc}} \right]^\alpha \right\}$$

Where, $\alpha = \epsilon_c * \frac{\epsilon_{cc}}{f_{cc}}$

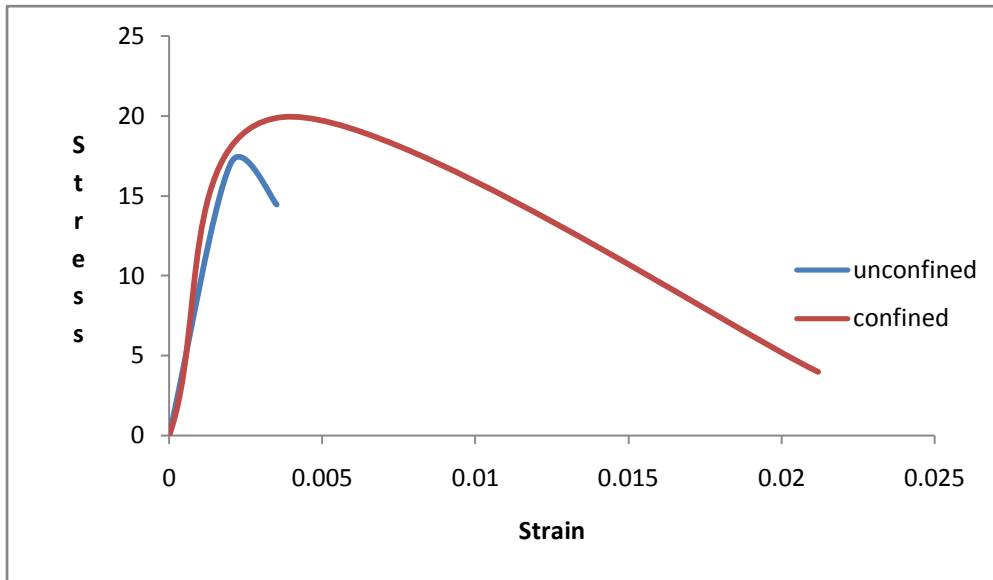
$$\epsilon_c: 3320\sqrt{f_{co}} + 6900$$

Descending Branch:

$$f_c = f_{cc} * e^{k_3(\epsilon_c - \epsilon_{cc})^{k_4}}$$

Where, $k_3 = \frac{\ln 0.5}{\epsilon_{cu} 50 - \epsilon_{cc}}$ and $k_4 = 0.3 + 12 * \frac{P_e}{f_{co}}$

2.3) Saaticioglu and Razvi Model (1992) for confined concrete:



Graph 2.3. Stress-Strain Curve

for ascending, $\sigma_c = f_{cc} * \left\{ \frac{2\epsilon_{co}}{\epsilon_{cc}} - \left(\frac{\epsilon_{co}}{\epsilon_{cc}} \right)^2 \right\}^{\frac{1}{1+2\vartheta}} \leq f_{cc}$

$$\epsilon_{cc} = \epsilon_{co} (1 + 5\vartheta)$$

$$\vartheta = \frac{k_1 * \sigma_{2e}}{k_3 * f_{ck}}$$

for descending, $\sigma_c = f_{cc} + \left(\frac{f_{cc} - 0.85f_{cc}}{\epsilon_{cc} - \epsilon_{uo}} \right) (\epsilon_c - \epsilon_{co})$

$$\varepsilon_{cu85} = 260 * \rho * \varepsilon_{cc} + \varepsilon_{co}$$

$$\rho = \frac{\epsilon A_{oxy} \sin \alpha}{S(b_{kx} + b_{ky})}$$

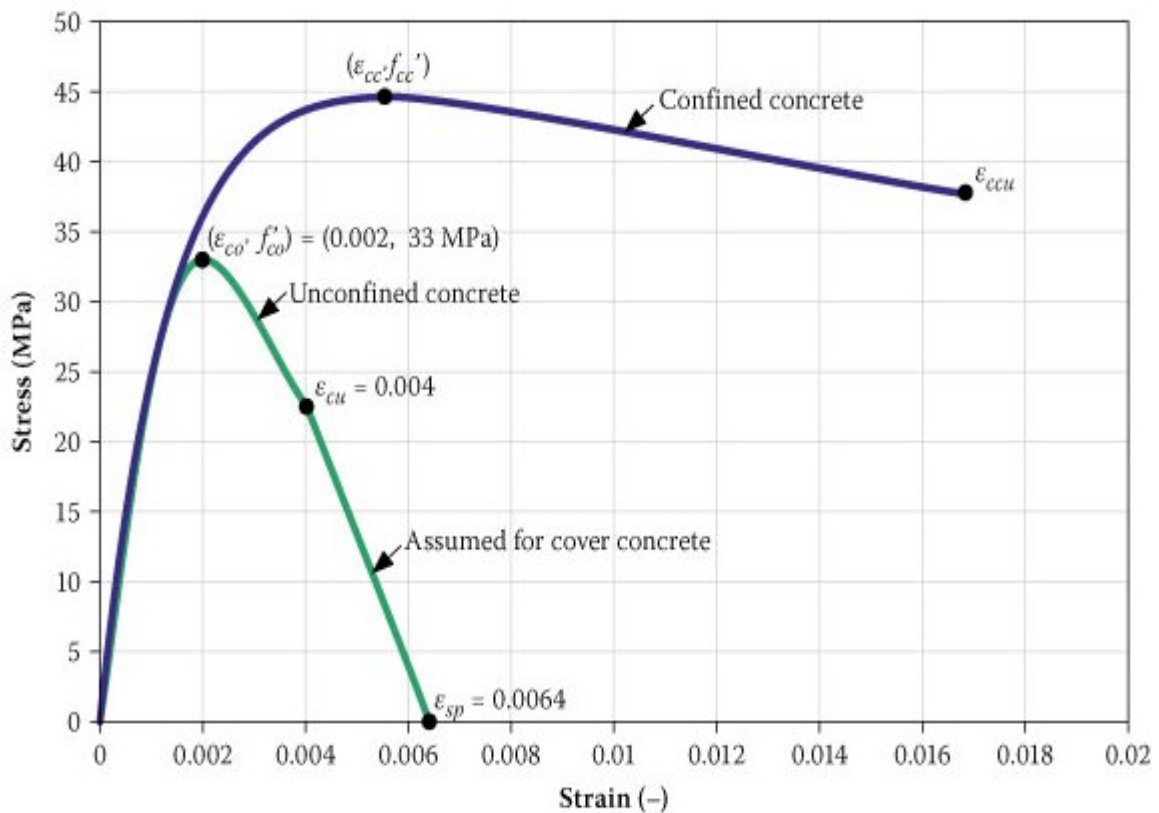
$\varepsilon_{co}, \varepsilon_{co}$ are taken as 0.002, 0.0035

$\varepsilon_{cc}, \varepsilon_{cu85}$ are the strains corresponding to peak stress and 85% of peak stress.

2.4) Modified Mander's and Fardis et al. (2001):

Advantages of Mander's Model:

- A single equation defines both the ascending and descending branches of stress strain curve.
- Model can also be used for unconfined concrete.
- Model can be applied to any shape of concrete member section confined by any kind of transverse reinforcement.



Chapter-3

THEORY AND FORMULATIONS

3. THEORY AND FORMULATIONS.

3.1. Theory:

Ductility is a desirable property of the reinforced concrete structures to ensure structural integrity in avoiding brittle failure during flexure. The ductile behavior of structure can be achieved by allowing the plastic hinges position at appropriate locations of the structural frame. These plastic hinges are designed to give adequate ductility to resist the structural collapse after yield strength of the material has been achieved. Based on the shape of the moment-curvature diagrams the available ductility can be found out.

Ductility can be defined as the capacity to undergo deformations without a considerable change in the flexural capacity of the member. The Ductility of a section can be expressed in the form of Curvature Ductility. The Curvature Ductility is given by,

$$\mu_{\phi} = \frac{\phi_u}{\phi_y}$$

Where ϕ_u is the curvature at ultimate when the concrete compression strain reaches specified limiting value, ϕ_y is the curvature when the tension reinforcement first reaches the yield strength. The definition of ϕ_y shows the influence of the yield strength of reinforcement steel on the calculation of μ_{ϕ} , while the definition of ϕ_u reflects the effect of ultimate strain of concrete in compression.

3.2 Introduction to OPENSEES:

The modelling of the structure is done in *Opensees* (Open System for Earthquake Engineering Simulation) which is an object oriented open-source software framework used to model structural and geotechnical systems and simulate their earthquake response. *Opensees* is primarily written in C++ and uses some FORTRAN and C numerical libraries for linear equation solving, and material and element customs. *Opensees* has progressive capabilities for modelling and analyzing the nonlinear response of systems using a wide range of material models, elements, and solution algorithms. It is an open-source; the website provides information about the software architecture, access to the source code, and the development

process. The open-source movement allows earthquake engineering researchers and users to build upon each other's accomplishments using *OpenSees* as community-based software. Another advantage of using *OpenSees* is that modelling frames with different sets of input variables can be done with the help of loops, whereas in conventional software's each case will have to be modelled separately.

Fiber Section modelling of element is done according to Spacone *et.al*, 1996 which can be employed using predefined command "section Fiber" in *OpenSees*.

3.3 Analysis of Various Confinement Models:

Various confinement models have been analyzed in *OpenSees* (Open System for Earthquake Engineering and Simulation). Confinement Models of beams with same cross-section with different spacing between stirrups of 2-legged and 3-legged are modelled and analyzed.

- i) f_{pc} : Concrete compressive strength at 28 days
- ii) $epsc0$: Concrete Strain at maximum strength: $epsc0$
- iii) f_{pcu} : Concrete crushing strength
- iv) $epsU$: Concrete strain at crushing strength

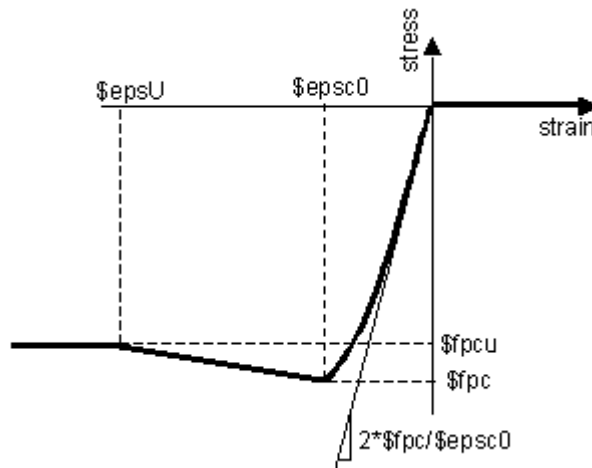


Figure 3.1 Parameters for *OpenSees*

Above mentioned four parameters are required for both cover concrete and core concrete. These values can be calculated by the various confined models mentioned in literature review.

Properties of reinforcing steel are given by,

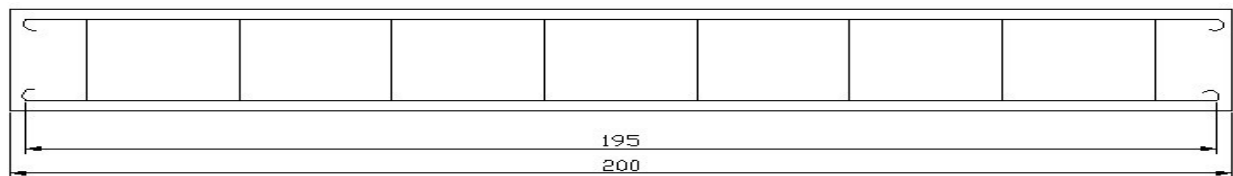
- i) Yield strength of reinforcing steel
- ii) Young's Modulus.

Parameters like cover dimension, area of steel in compression and area of steel in tension also required to analyze the moment-curvature of particular section.

The drawings of various confinement models with 2-legged and 3-legged stirrups are given below.

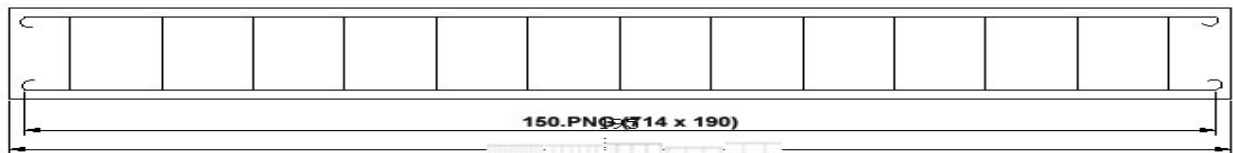
Case (I):

Beam with stirrup spacing @ 250mm c/c



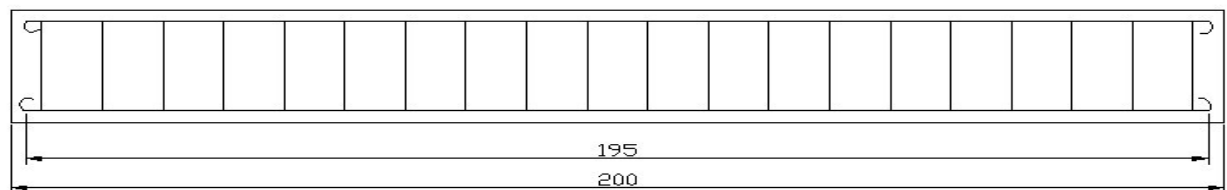
Case (II):

Beam with stirrup spacing @ 150mm c/c

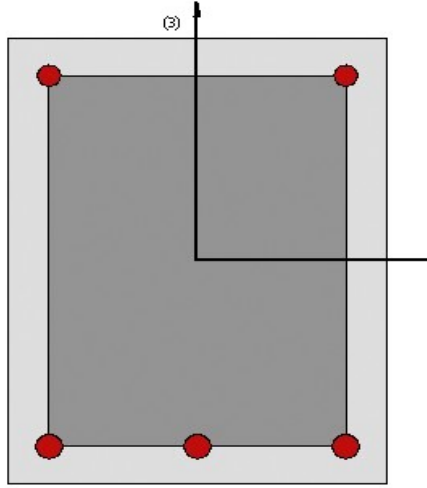


Case (III):

Beam with stirrup spacing @ 100mm c/c



The Beam cross section for analysis is 230mm x 300 mm with 10 mm diameter hook bars in compression side and three 12 mm diameter main bars in tension side with a clear cover of 25 mm on all sides.



Design parameters for analysis:

Serial Number	2-Legged (N/mm ²)	3-Legged (N/mm ²)
1. Concrete Strength	21.9	23
2. Grade of Steel	415	415

Models for confined concrete:

3.4. Hong K N and Han S H Model (2005):

$$\text{Confining Pressure : } P_e = k_e \rho_w f_{ys}$$

$$k_e = \frac{\left[1 - \epsilon \frac{w_i^2}{6b_c d_c}\right] \left[1 - \frac{S'}{2b_c}\right] \left[1 - \frac{S'}{2d_c}\right]}{1 - \rho_{cc}}$$

$$f_{ys} = \text{grade of tie}$$

$$\rho_w = \frac{\epsilon A_{oxy}}{S(b_{kx} + b_{ky})}$$

$$\rho_{cc} = \text{Longitudinal bar ration in core section}$$

$$S' = \text{tie spacing}$$

$$b_c, d_c = \text{core diamenssions}$$

$$\frac{f_{cc}}{f_{co}} = 1 + 1.6 \left(\frac{p_e}{f_{co}}\right)^{0.5}$$

$$\varepsilon_{cu50} = \varepsilon_{uo} + 30 * \frac{k_2^2 * P_e}{f_{co}^2}$$

$$f_{co} = 0.85 f_{ck}$$

$$k_2 = 1$$

Stress-Strain Curve Equations for ascending and descending branches

$$f_c = f_{cc} \{1 - [1 - \frac{\varepsilon_c}{\varepsilon_{cc}}]^\alpha\}$$

$$\text{Where, } \alpha = \varepsilon_c * \frac{\varepsilon_{cc}}{f_{cc}}$$

$$\varepsilon_c : 3320 \sqrt{f_{co}} + 6900$$

$$f_c = f_{cc} * e^{k_3(\varepsilon_c - \varepsilon_{cc})^{k_4}}$$

$$\text{Where, } k_3 = \frac{\ln 0.5}{\varepsilon_{cu50} - \varepsilon_{cc}} \text{ and } k_4 = 0.3 + 12 * \frac{P_e}{f_{co}}$$

In Above

Equations, f_{cc} , f_{co} , ε_{cc} and ε_{cu50} are representing Peak stresses for confined and unconfined concrete and strains corresponding to peak stress and 50% of peak stress.

3.5. Saatcioglu and Razvi.et.al Model (1992):

$$f_{cc} = k_3 * f_{ck} + k_1 * \sigma_{2e}$$

$$\text{Where, } k_1 = \frac{6.7}{\sigma_{2e}^{0.17}}$$

$$k_3 = 0.85 \text{ for normal strength concrete}$$

$$\sigma_{2e} = \frac{\sigma_{2ex} * b_{kx} + \sigma_{2ey} * b_{ky}}{b_{kx} + b_{ky}}$$

$$\sigma_{2ex} = \beta_x * \sigma_{2x}$$

$$\sigma_{2ey} = \beta_y * \sigma_{2y}$$

$$\sigma_{2x} = \frac{A_{ox} f_{yk} \sin \alpha}{S * b_{kx}}$$

$$\sigma_{2y} = \frac{A_{oy} f_{yk} \sin \alpha}{S * b_{ky}}$$

$$\beta_x = 0.26 * \sqrt{\frac{b_{kx}^2}{a_x * S * \sigma_{2x}}}$$

$$\beta_y = 0.26 * \sqrt{\frac{b_{ky}^2}{a_y * S * \sigma_{2y}}}$$

$\in A_{ox} f_{yk} \sin \alpha$: Summation of cross – section in x – direction

$\in A_{oy} f_{yk} \sin \alpha$: Summation of Cross – sections in y – direction

a_x, a_y : Unsupported length of ties in x, y – directions

b_{kx}, b_{ky} : Core diamenssions of rectangular cross – section

f_{yk} : Yield strength of tie

S : spacing of ties

α : inclination of legs in respective direction

$$\text{for Ascending portion, } \sigma_c = f_{cc} * \left\{ \frac{2\varepsilon_{co}}{\varepsilon_{cc}} - \left(\frac{\varepsilon_{co}}{\varepsilon_{cc}} \right)^2 \right\}^{\frac{1}{1+2\vartheta}} \leq f_{cc}$$

$$\varepsilon_{cc} = \varepsilon_{co} (1 + 5\vartheta)$$

$$\vartheta = \frac{k_1 * \sigma_{2e}}{k_3 * f_{ck}}$$

$$\text{for descending portion, } \sigma_c = f_{cc} + \left(\frac{f_{cc} - 0.85f_{cc}}{\varepsilon_{cc} - \varepsilon_{uo}} \right) (\varepsilon_c - \varepsilon_{co})$$

$$\varepsilon_{cu85} = 260 * \rho * \varepsilon_{cc} + \varepsilon_{co}$$

$$\rho = \frac{\epsilon A_{oxy} \sin \alpha}{S(b_{kx} + b_{ky})}$$

$\varepsilon_{co}, \varepsilon_{co}$ are taken as 0.002, 0.0035

$\varepsilon_{cc}, \varepsilon_{cu85}$ are the strains corresponding to peak stress and 85% of peak stress.

3.6. Mander's Model (2001):

The Equation of the Stress-Strain Profile as per this model is given by,

$$f'_c = \frac{f'_{cc} * x * r}{r - 1 + x^r}$$

“x” Value is given by,

$$x = \frac{\epsilon'_c}{\epsilon'_{cc}}$$

And “r” is given by

$$r = \frac{E_c}{E_c - E_{sec}}$$

Where E_{sec} is Secant modulus of Elasticity of Concrete

$$E_{sec} = \frac{f'_{cc}}{\epsilon'_{cc}}$$

Equation for strain at Yield Stress is,

$$\epsilon'_{cc} = \left[\left(\frac{f'_{cc}}{f'_{co}} - 1 \right) 5 + 1 \right] * \epsilon'_{co}$$

Equation for Strain at Ultimate Stress is,

$$\epsilon'_{cu} = \frac{(0.6 * \rho_s * f_{yh} * \epsilon_{sm})}{f'_{cc}} + 0.004$$

Then, the Equation for Stress at yield point is,

$$f'_{cc} = f'_{co} \left\{ 1 + 3.7 \left[\frac{0.5 * \rho_s * f_{yh} * K_e}{f'_{co}} \right]^{0.85} \right\}$$

Chapter-4

ANALYTICAL RESULTS AND DISCUSSIONS

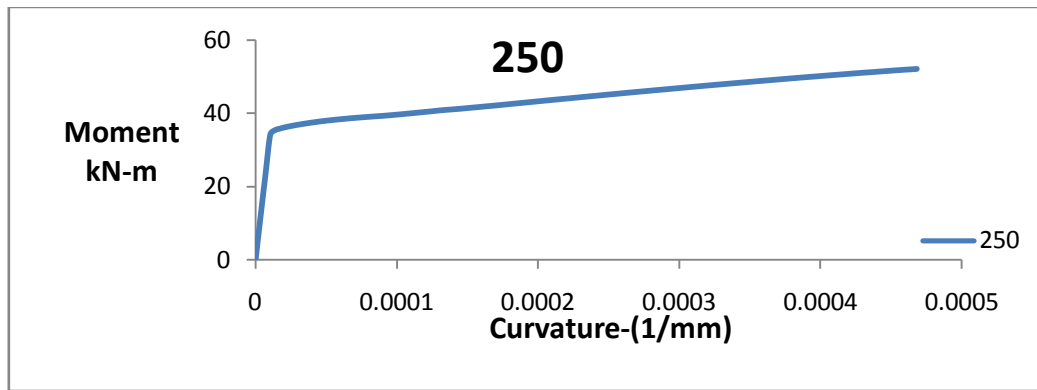
4. ANALYTICAL RESULTS AND DISCUSSIONS.

To plot the curve of Moment vs. Curvature the analysis is stopped where the section reaches maximum strain as per confinement model. The stress- strain values of particular section can be obtained by using stress-strain recorder in analysis output part.

4.1. Moment vs. Curvature (*HONG K N and Han S H (2005) Model*):

4.1.1. Beam with two legged stirrups @ 250mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 250mm c/c and shown in graph 4.1.



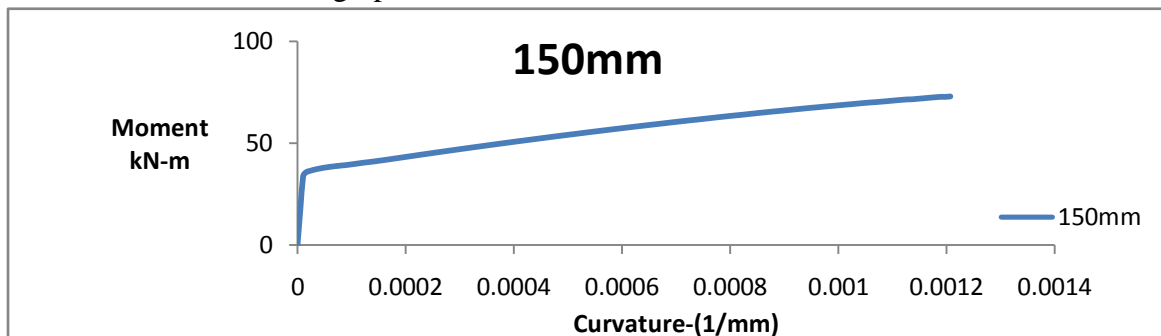
Graph 4.1 Moment vs. Curvature for Beam 1.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{\phi_u}{\phi_y}$$

$$\text{i.e. } \frac{0.000468}{1.39 \times 10^{-5}} = 33.69$$

4.1.2. Beam with two legged stirrups @ 150mm c/c spacing:

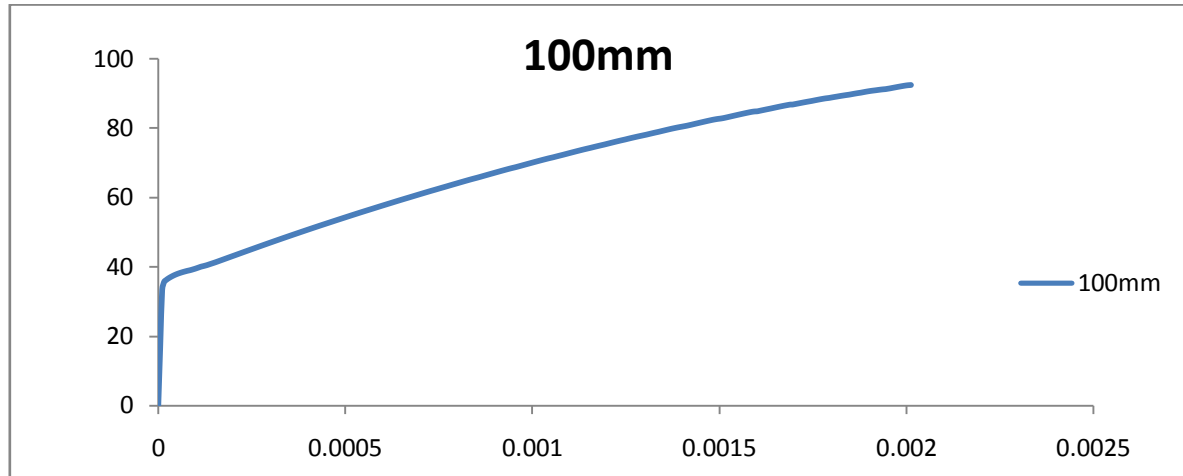
A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 150mm c/c and shown in graph 4.2.



$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.001207}{1.39 \times 10^{-5}} = 86.83$$

4.1.3. Beam with two legged stirrups @ 100mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 100mm c/c and shown in graph 4.3.

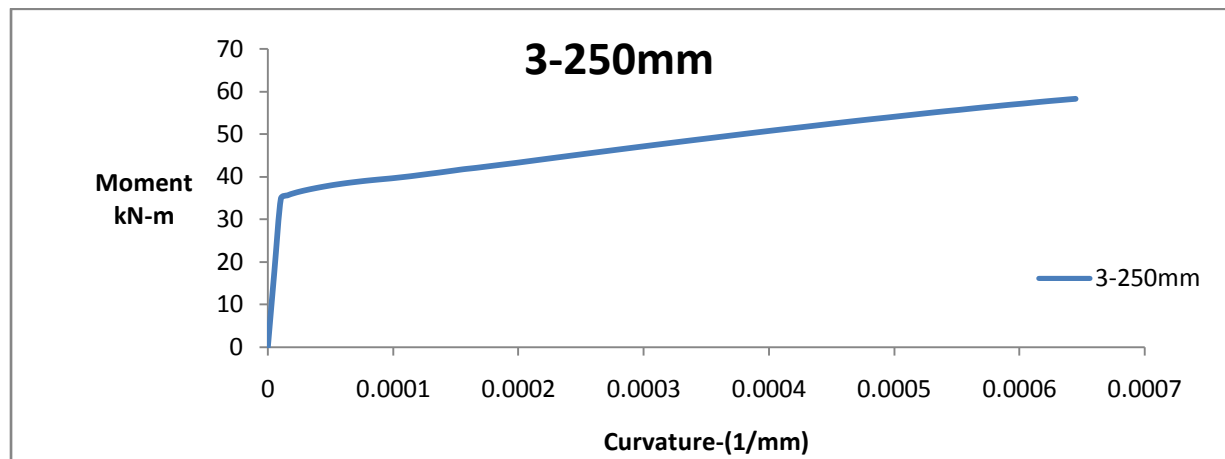


Graph 4.3 Moment vs. Curvature for Beam 3.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.002013}{1.39 \times 10^{-5}} = 144.82.$$

4.1.4. Beam with three legged stirrups @ 250mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 250mm c/c and shown in graph 4.4.

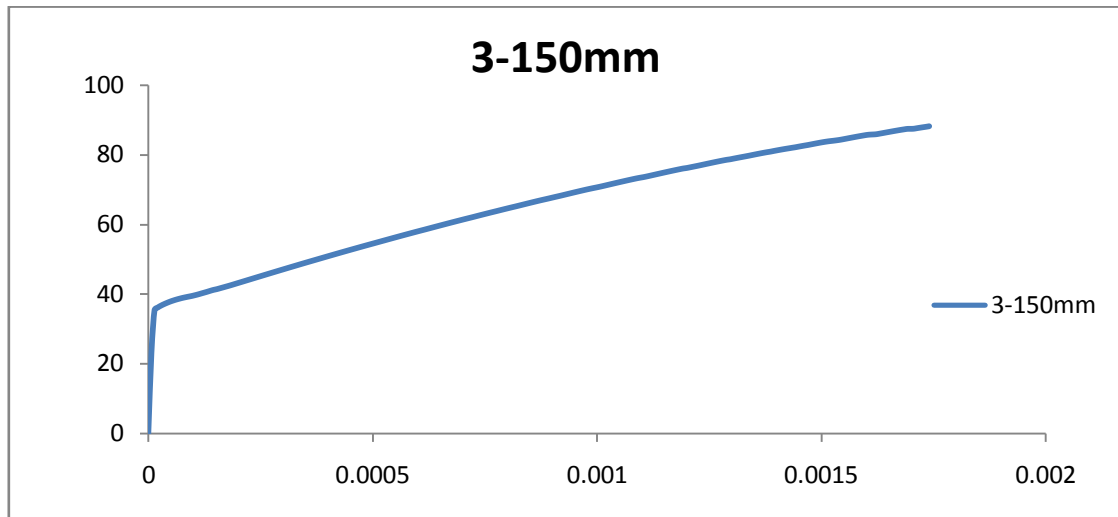


Graph 4.4 Moment vs. Curvature for Beam 4.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.000645}{1.56 \times 10^{-5}} = 41.34.$$

4.1.5. Beam with three legged stirrups @ 150mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 150mm c/c and shown in graph 4.5.

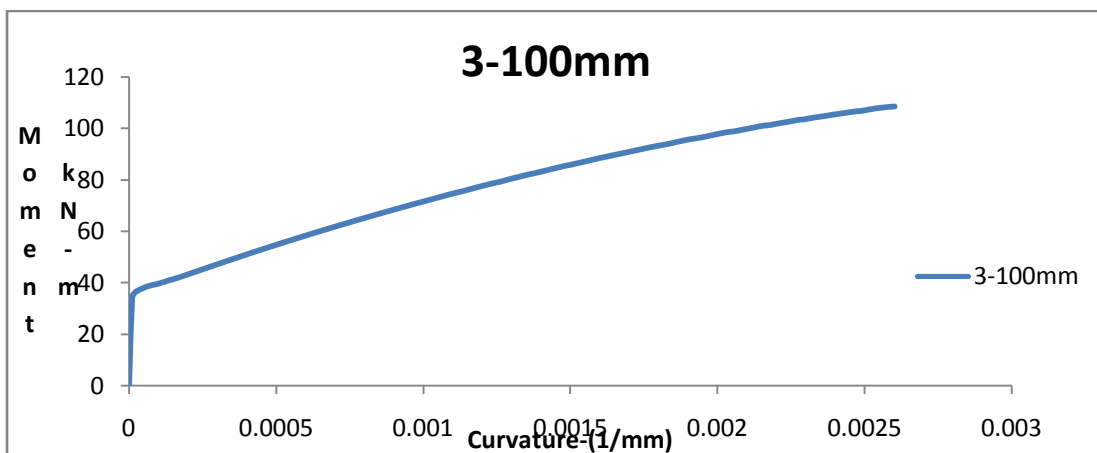


Graph 4.5 Moment vs. Curvature for Beam 5.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.00174}{1.56 \times 10^{-5}} = 111.54$$

4.1.6. Beam with three legged stirrups @ 100mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 100mm c/c and shown in graph 4.6.



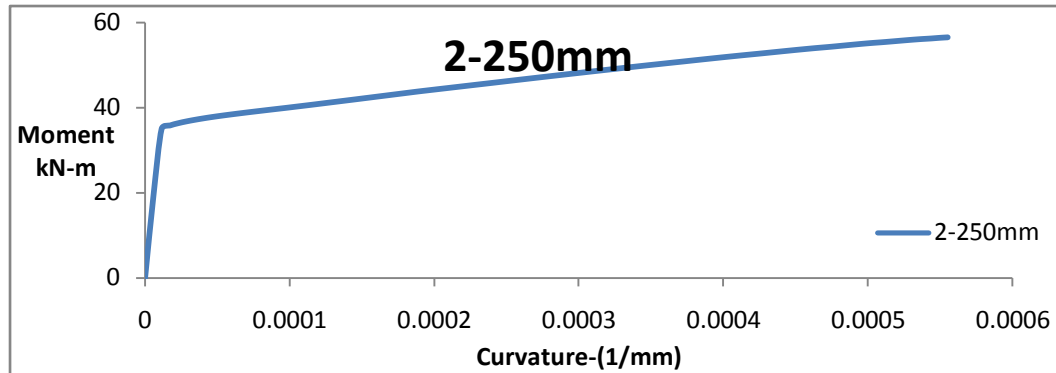
Graph 4.6 Moment vs. Curvature for Beam 6.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.0026}{1.56 \times 10^{-5}} = 166.67$$

4.2. Moment vs. Curvature (*Saatcioglu and Raazvi (1992) Model.*)

4.2.1. Beam with two legged stirrups @ 250mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 250mm c/c and shown in graph4.7.

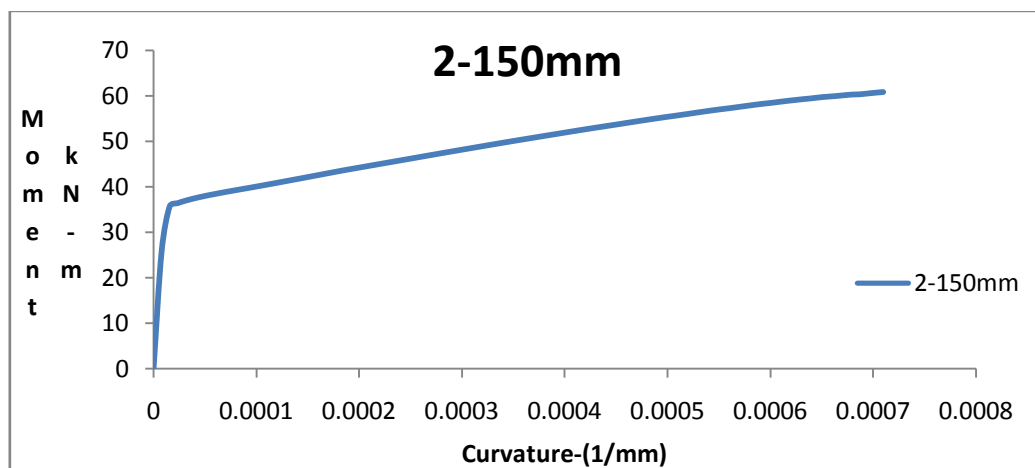


Graph 4.7 Moment vs. Curvature for Beam 1.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.000555}{1.72 \times 10^{-5}} = 32.27$$

4.2.2. Beam with two legged stirrups @ 150mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 150mm c/c and shown in graph4.8

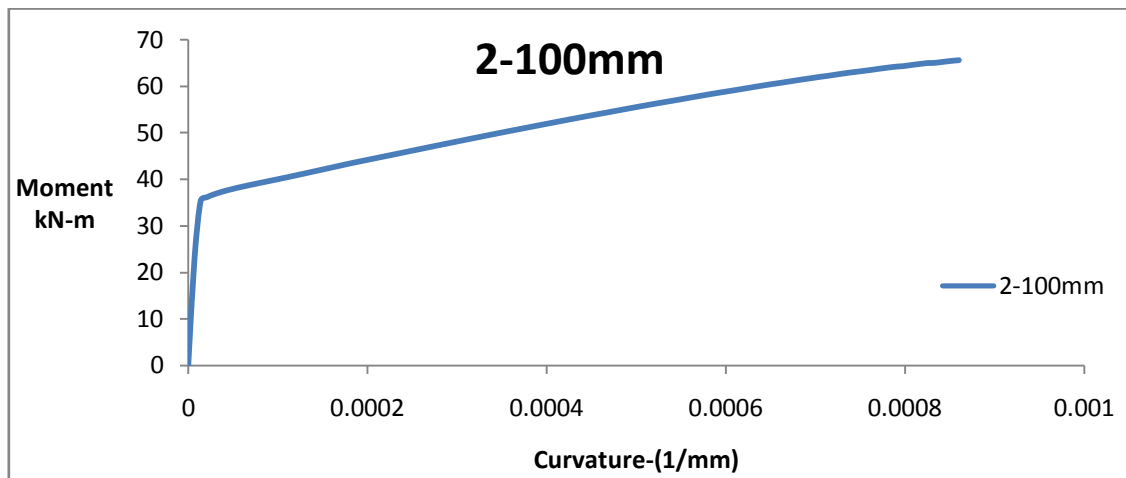


Graph 4.8 Moment vs. Curvature for Beam 2.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.00071}{1.72 \times 10^{-5}} = 41.30$$

4.2.3. Beam with two legged stirrups @ 100mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 2-legged stirrups @ 100mm c/c and shown in graph4.9

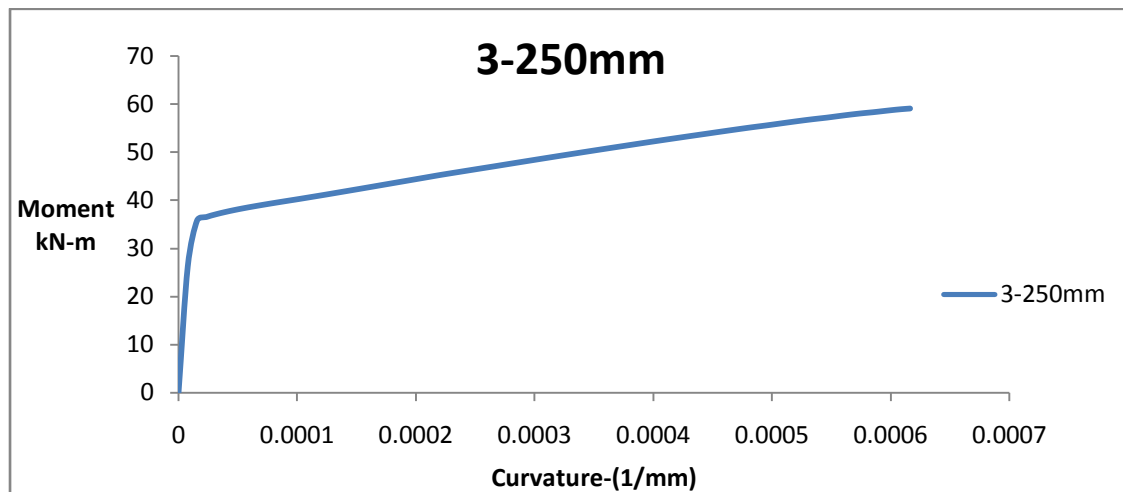


Graph 4.9 Moment vs. Curvature for Beam 3.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.00086}{1.72 \times 10^{-5}} = 50.00$$

4.2.4. Beam with three legged stirrups @ 250mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 250mm c/c and shown in graph4.10

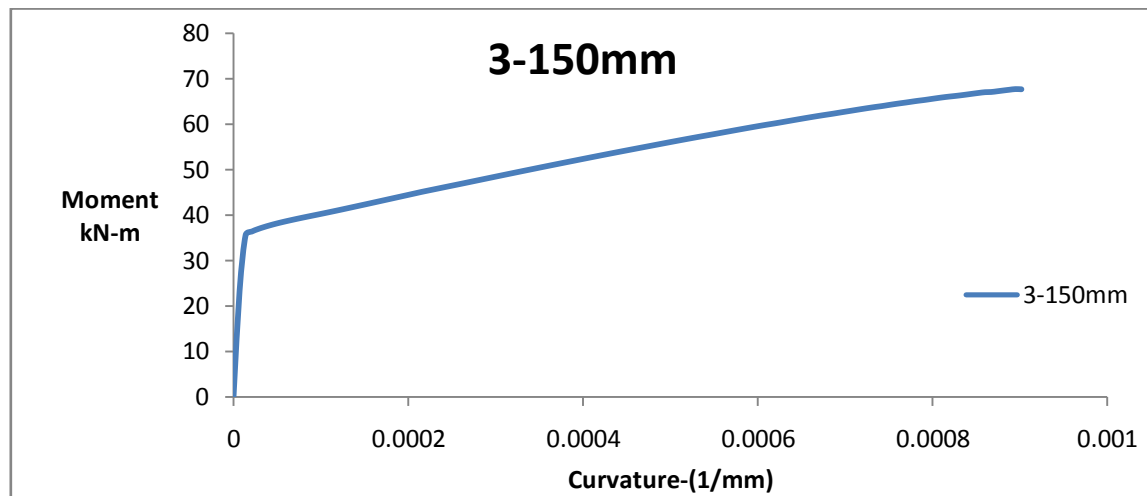


Graph 4.10 Moment vs. Curvature for Beam 4.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.000616}{1.8 \times 10^{-5}} = 34.23$$

4.2.5. Beam with three legged stirrups @ 150mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 150mm c/c and shown in graph4.11

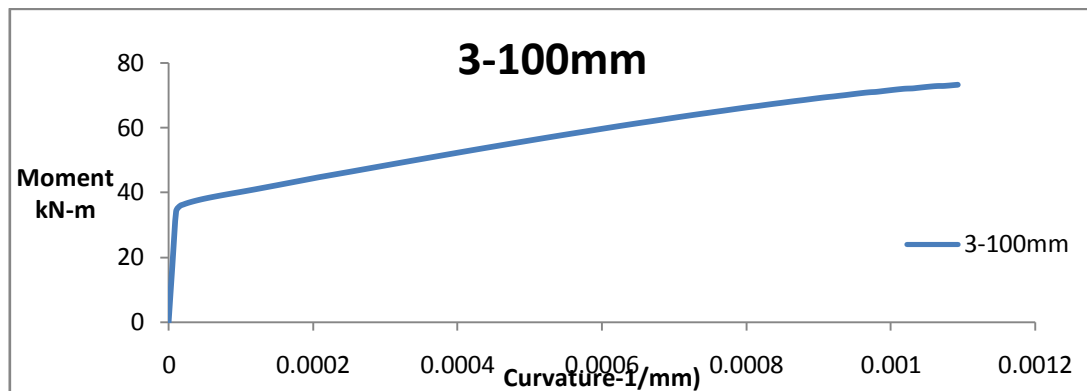


Graph 4.11 Moment vs. Curvature for Beam 5.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.000894}{1.8 \times 10^{-5}} = 49.67$$

4.2.6. Beam with three legged stirrups @ 100mm c/c spacing:

A Graph is plotted between moment vs. curvature for beam with 3-legged stirrups @ 100mm c/c and shown in graph4.12.



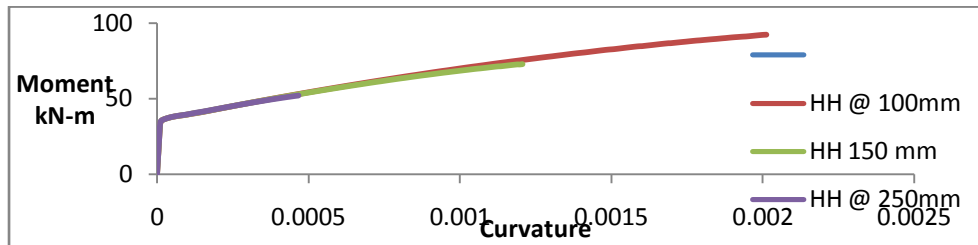
Graph 4.12 Moment vs. Curvature for Beam 6.

$$\text{Curvature Ductility } (\mu_{\phi}) = \frac{0.0011}{1.8 \times 10^{-5}} = 61.12.$$

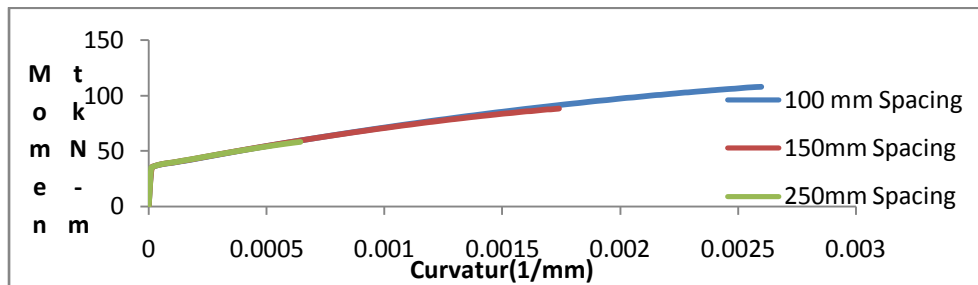
4.3. Comparison of Results:

In this section the analytical results are compared between both the models with 2-legged, 3-legged Stirrups and with different spacing of Stirrups.

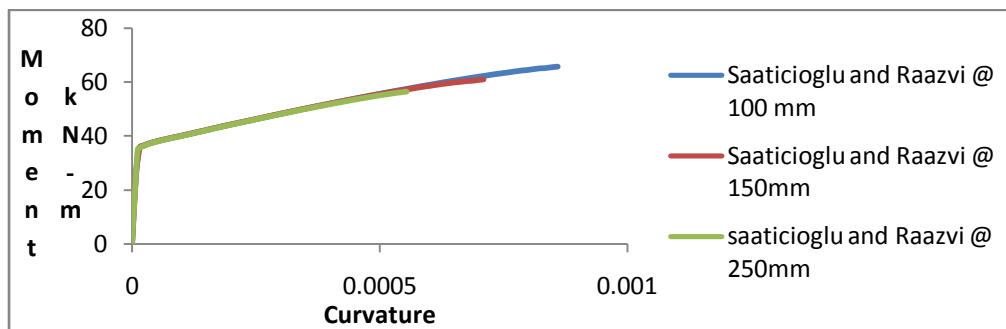
1) 2-legged Beams



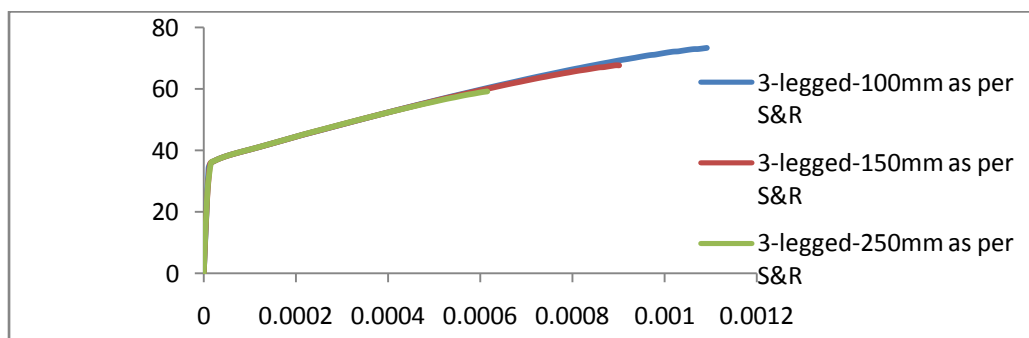
2) 3-legged Beams



3) 2-legged Beams



4) 3-legged beams



S.No.	Beam Description and Spacing	HONG K N and HAN S H (2005) Model (μ_ϕ)	Saatcioglu and Razvi (1992) Model (μ_ϕ)
1	2-Legged-250mm	33.69	32.27
2	2-legged-150mm	86.83	41.30
3	2-legged-100mm	144.82	50
4	3-legged-250mm	41.34	34.23
5	3-legged-150mm	111.54	49.67
6	3-legged-100mm	166.67	61.12

Table 4.1 Comparison of analytical models

4.4. Discussions:

- ✓ The results given in Table indicate that as per *Hong K N and Han S H* Model there is drastic increase in curvature ductility when the stirrup spacing decreases and not much increase in curvature ductility when numbers of stirrup legs are increased. Same trend is obtained by following *Saatcioglu and Razvi* model.
- ✓ The percentage of increase in ductility is more following *Hong K N and Han S H* Model than *Saatcioglu and Razvi* model in either case when stirrup spacing is decreased or no of legs of stirrup is increased.
- ✓ Both the models are giving almost similar initial ductility.

Chapter-5

EXPERIMENTAL SETUP

5. EXPERIMENTAL SETUP:

5.1. Material Properties

5.1.1. Concrete:

A mix of concrete of M20 grade is designed by using Portland Slag cement of Konark brand , locally available sand confirming to Zone III and 20 mm down size aggregate for a slump of 30mm. The mix is designed following IS 10262-1988.

The proportion of design mix adopted for the experiment is 1:1.7:3.8 by weight and water cement ratio is taken as 0.6.

Table 5.1 Design Mix Proportion of Concrete

Description	cement	Fine aggregate	Coarse aggregate	Water
Mix proportion	1	1.7	3.8	0.6

Table 5.2 Test Result of specimens after 28 Days

S.No.	Beams	Cube Compressive Strength (N/mm^2)	Cylinder Compressive Strength (N/mm^2)
1	2-Legged-250mm	18	16.4
2	2-legged-150mm	24	21.2
3	2-legged-100mm	23.7	21.7
4	3-legged-250mm	23.6	22
5	3-legged-150mm	18.5	16.9
6	3-legged-100mm	26.9	20

5.1.2. Reinforcing Steel:

Steel bars of Fe415 grade of 8mm, 10mm and 12mm diameter are used for reinforcement. All bars are tested for Tensile strength and they comply with the code IS 1786-1985.

Table 5.3 Tensile Strength of reinforcing steel bars

SI no of the sample	Diameter of the bar tested in mm.	0.2% proof stress (yield strength)N/mm ²	Average yield strength N/mm ²
1	8	524	523
2	8	522	
3	10	535	533.5
4	10	532	
5	12	590	580
6	12	570	

5.2. Casting of Specimens:

For the investigation six beams are cast. All beams are of same cross section 230mm x 300 mm, provided with 2 main bars of 12 mm diameter on tension side and 2 hook bars of 10 mm on compression side. Vertical stirrups of 8 mm diameter with varying spacing and no. of legs are provided. Spacing adopted are 250,150 and 100 mm c/c with 2 legged and 3 legged stirrups. All beams are designed to fail in flexure.

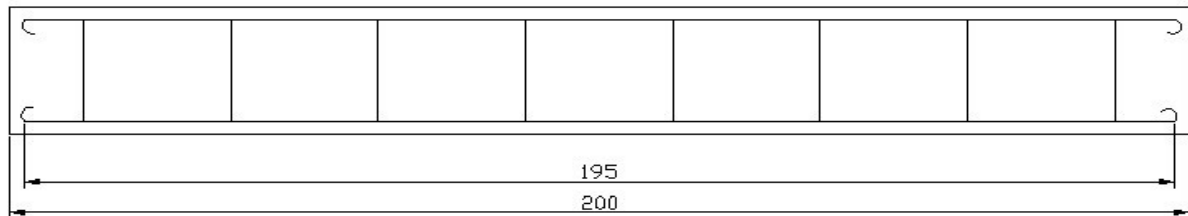


Fig.5.2.1.Beam1 (Two legged) & Beam 4(Three legged) with stirrups @ 250mm c/c spacing.

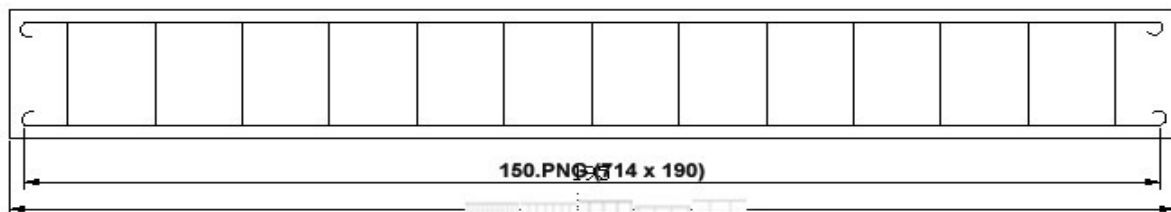


Fig.5.2.2. Beam2 (Two legged) & Beam5 (Three legged) with stirrups @ 150mm c/c spacing.

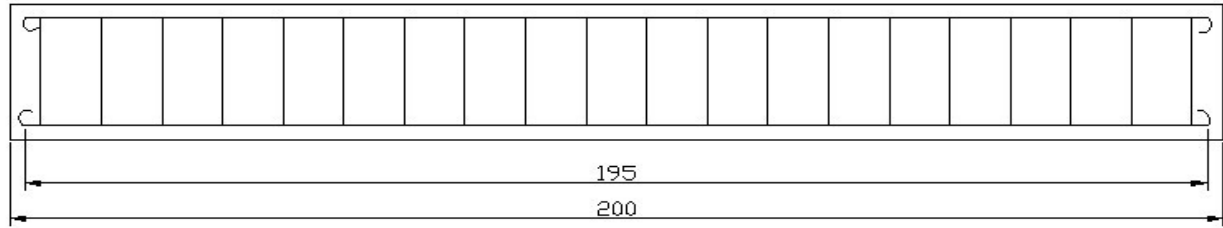


Fig.5.2.3 .Beam3 (Two legged) &Beam 6 (Three legged) with stirrups @ 100mm c/c spacing.

Beams are cast in rectangular moulds. These moulds are removed after 24 hrs and also the beams are taken out and cover with jute bags for curing for 28 days. Along with beam standard specimens are cast to get the properties of the concrete, these include 3 numbers of cubes and 3 numbers of cylinders. These are tested for cubical compressive strength (f_{ck}) and cylindrical compressive strength (f_c).

5.3. Tests and Results:

All the six beams are tested till the complete failure under monotonically increased static single point load applied at the centre of the beam in statistical state. Three dial gauges are placed along the length of the beam to measure the deflections.

5.3.1. Measurement of Strain:

For measuring stain in the beam two points are marked on both sides of the centre line along the length of the beam. These points are marked in both compression and tension zones with cover of 25mm from top and bottom levels of the beam. The initial length between two points in each is 100mm and the distance between Markings of compression and tension zone is 250mm. While applying load, for every 10 kN increase in load the length between either sides of the points are measured by using mechanical strain gauge in compression and tension zones and strains are calculated at each increment of loading.

5.3.2. Calculation of Curvature:

After getting strains in both zones, curvatures are calculated. The strains in compression and tension are combined to get the resultant strain. The ration of resultant strain to the lever arm will be the curvature. *Slope of Strain Diagram* is Curvature.

5.3.3. Beam-1 (Two legged stirrups at 250mm c/c spacing):



Figure 5.1 Beam 1

First Crack was observed at 50kN and beam failed in flexure at 120kN.

Table 5.4

S.No.	Moment (<i>kN-m</i>)	Tension zone Readings (<i>mm</i>)	Compression zone Readings (<i>mm</i>)	Strain $\times 10^{-3}$	Curvature (1/ <i>mm</i>) $\times 10^{-6}$
1	0	100	100	0	0
2	5	100	100	0	0
3	10	100.07	100.03	1	4
4	15	100.09	100.03	1.2	4.8
5	20	100.13	100.05	1.8	7.2
6	25	100.61	100.18	7.9	31.6
7	30	101.00	100.23	12.3	49.2
8	35	102.09	100.26	23.5	94
9	40	103.03	100.37	34	136
10	45	104.65	100.45	51	204
11	50	105.79	100.48	62.7	250.8
12	55	106.87	100.56	74.3	297.2

5.3.4. Beam-2 (Two legged stirrups at 150mm c/c spacing):

First Crack was observed at 75kN and beam failed in flexure at 130kN.

Figure 5.2



Table 5.5

S.No.	Moment (<i>kN-m</i>)	Tension zone Readings (<i>mm</i>)	Compression zone Readings (<i>mm</i>)	Strain $\ast 10^{-3}$	Curvature (1/ <i>mm</i>) $\ast 10^{-6}$
1	0	100	100	0	0
2	5	100.01	100	0.1	0.4
3	10	100.06	100	0.6	2.4
4	15	100.09	100.02	1.1	4.4
5	20	100.21	100.04	2.5	10
6	25	100.56	100.05	6.1	24.8
7	30	101.29	100.15	14.4	57.6
8	35	102.20	100.20	24	94
9	40	104.58	100.31	48.9	195.6
10	45	105.73	100.50	62.3	249.2
11	50	106.90	100.73	76.3	305.2
12	55	108.20	100.95	91.5	366
13	60	111.62	101.17	127.9	511.6

5.3.5. Beam-3 (Two legged stirrups at 100mm c/c spacing):

First Crack was observed at 60kN and beam failed in flexure at 145kN.



S.No.	Moment (kN-m)	Tension zone Readings (mm)	Compression zone Readings (mm)	Strain *10⁻³	Curvature (1/mm) *10⁻⁶
1	0	100	100	0	0
2	5	100	100	0	0
3	10	100.03	100.00	0.3	1.2
4	15	100.07	100.01	0.8	2.8
5	20	100.11	100.03	1.4	5.6
6	25	100.36	100.06	4.2	16.8
7	30	100.84	100.12	9.6	38.4
8	35	101.76	100.37	21.3	85.2
9	40	102.83	100.61	34.4	137.6
10	45	106.75	100.85	76	304
11	50	108.24	101.07	93.1	372.4
12	55	109.85	101.65	114	456
13	60	113.73	102.05	157.8	631.2

Table 5.6

The Beam has completely collapsed in flexure. The breaking of tensile reinforcement below the load point has occurred which is clearly visible in the Figure 5.3.

5.3.6. Beam-4 (Three legged stirrups at 250mm c/c spacing):

First Crack was observed at 70kN and beam has partially failed in flexure at 135kN.



Figure 5.4

S.No.	Moment (<i>kN-m</i>)	Tension zone Readings (<i>mm</i>)	Compression zone Readings (<i>mm</i>)	Strain $\ast 10^{-3}$	Curvature (1/ <i>mm</i>) $\ast 10^{-6}$
1	0	100	100	0	0
2	5	100.04	100	0.4	0
3	10	100.08	100.02	1	4
4	15	100.14	100.05	1.9	7.6
5	20	100.21	100.06	2.7	5.6
6	25	101.35	100.14	14.9	59.6
7	30	102.68	100.29	29.7	118.8
8	35	103.91	100.45	43.6	174.4
9	40	105.26	100.68	59.4	237.6
10	45	106.30	100.93	72.3	389.2
11	50	107.20	101.27	84.7	338.8
12	55	107.93	101.54	94.7	378.8
13	60	109.85	101.77	116.2	464.8

Table 5.7

5.3.7. Beam-5 (Three legged stirrups at 150mm c/c spacing):

First Crack was observed at 80kN and beam has failed in flexure at 150kN.



Figure 5.5

S.No.	Moment (kN-m)	Tension zone Readings (mm)	Compression zone Readings (mm)	Strain *10 ⁻³	Curvature (1/mm) *10 ⁻⁶
1	0	100	100	0	0
2	5	100.03	100	0	1.2
3	10	100.09	100.01	1	4
4	15	100.13	100.04	1.7	6.8
5	20	100.36	100.12	4.8	19.2
6	25	100.47	100.16	6.3	25.2
7	30	100.76	100.23	9.9	39.6
8	35	100.95	100.37	13.2	52.8
9	40	101.84	100.44	23.8	95.2
10	45	103.97	100.58	45.5	182
11	50	107.33	100.74	80.7	322.8
12	55	110.69	101.27	119.6	478.4
13	60	114.10	101.52	157.8	624.8

Table 5.8

5.3.8. Beam-6 (Three legged stirrups at 100mm c/c spacing):

First Crack was observed at 75kN and beam has failed in flexure at 185kN. Multiple cracks in flexure zone were observed.



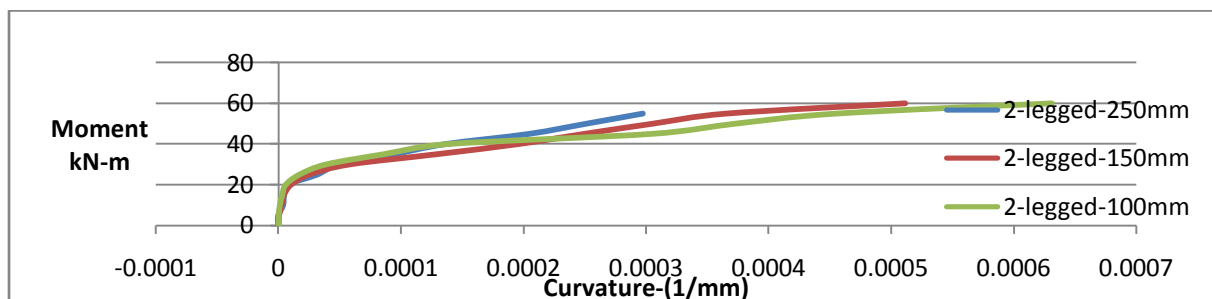
Figure 5.6



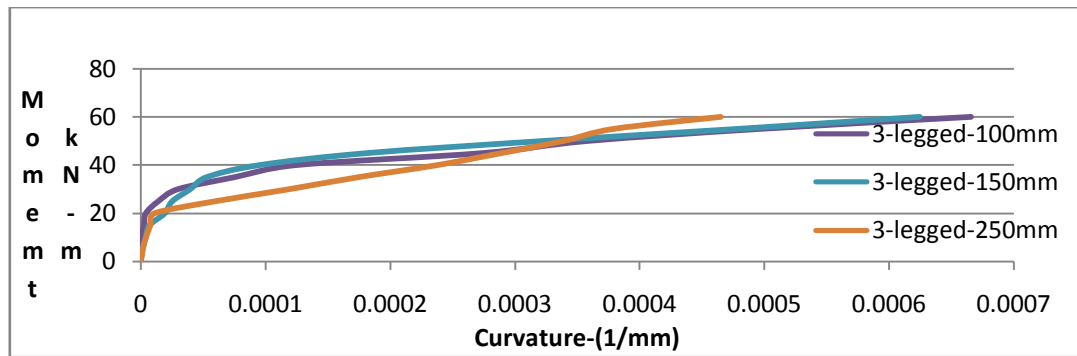
Figure 5.7

S.No.	Moment (<i>kN-m</i>)	Tension zone Readings (<i>mm</i>)	Compression zone Readings (<i>mm</i>)	Strain * 10^{-3}	Curvature (1/ <i>mm</i>) * 10^{-6}
1	0	100	100	0	0
2	5	100.02	100	0.2	0.8
3	10	100.02	100.00	0.2	0.8
4	15	100.04	100.02	1	4
5	20	100.08	100.02	1.4	5.6
6	25	100.32	100.03	3.5	14
7	30	100.69	100.05	7.4	29.6
8	35	100.78	100.1	18.8	75.2
9	40	102.76	100.35	31.1	124.4
10	45	106.45	100.41	68.6	274.4
11	50	108.40	100.55	89.5	358
12	55	111.72	100.68	124	496
13	60	115.27	101.37	166.4	665.6

Table 5.9



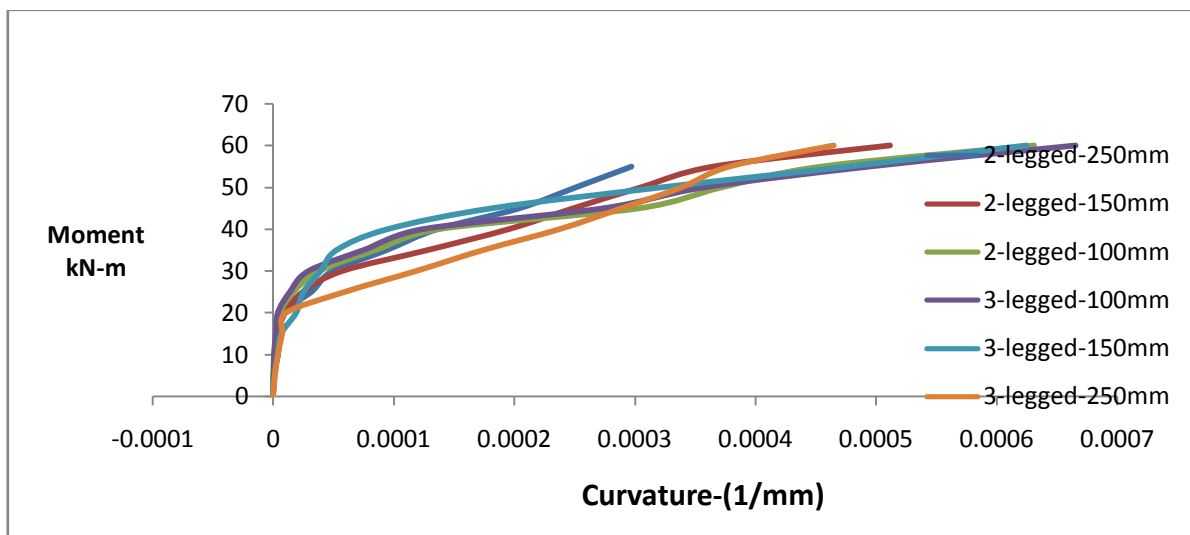
Graph 5.6 Moment vs. Curvature (Two legged-experimental)



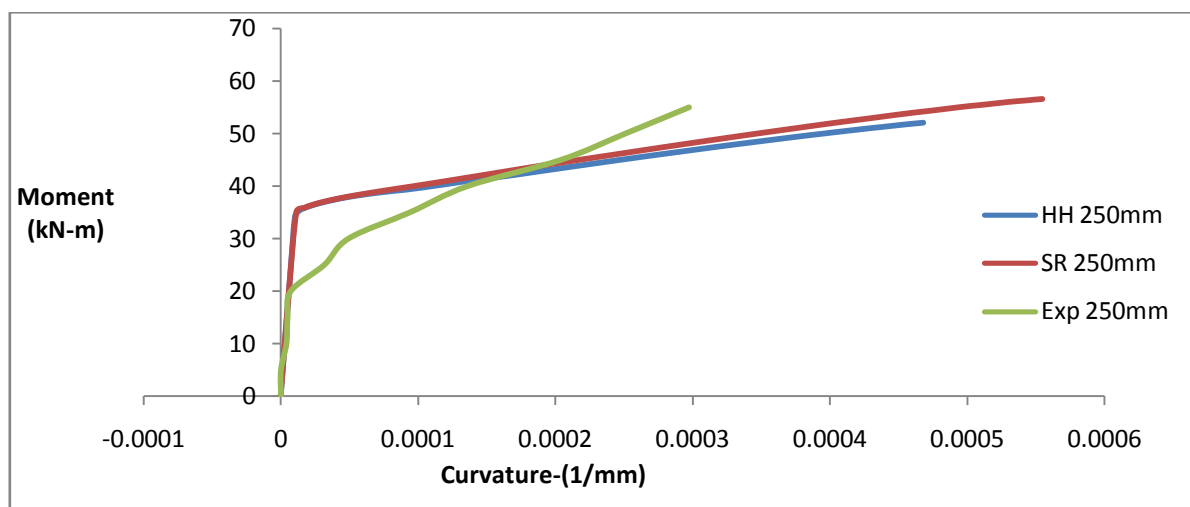
Graph 5.7 Moment vs. Curvature (Three legged-experimental)

5.4. Comparison of Results:

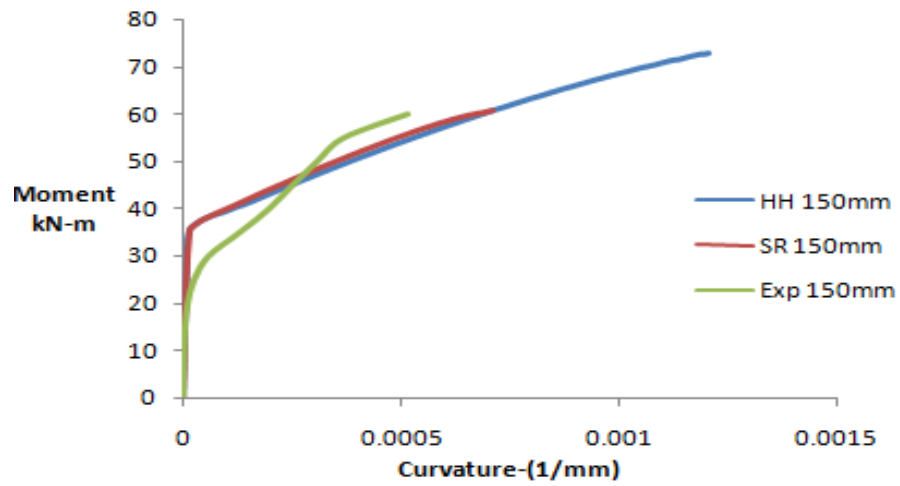
The experimental Moment vs. Curvature curves of all six beams are compared with the two analytical confinement models.



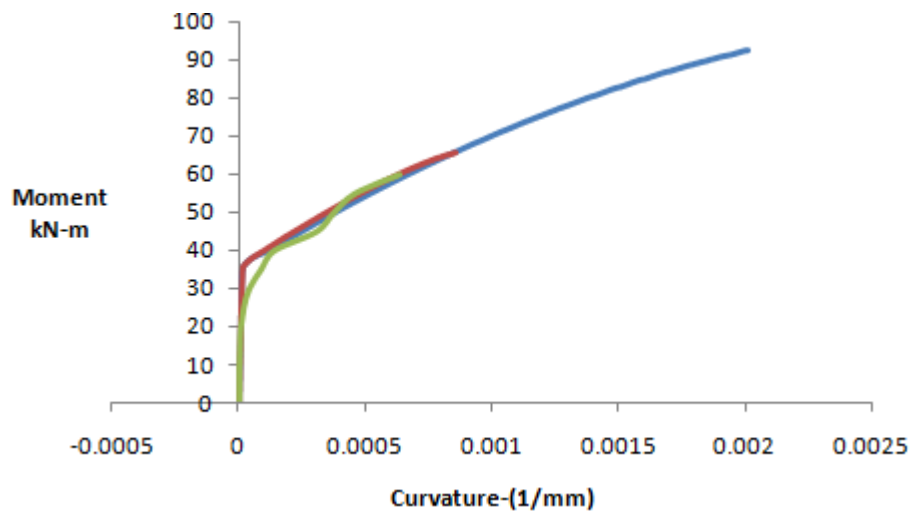
Graph 5.1 Experimental Moment vs. Curvature



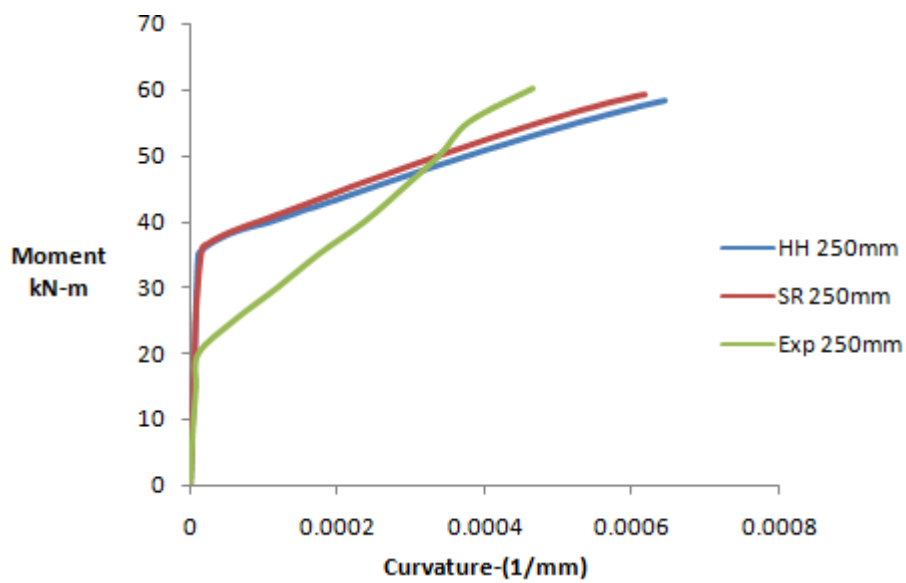
Graph 5.2 Comparison of experimental and analytical results for 2-legged 250mm c/c



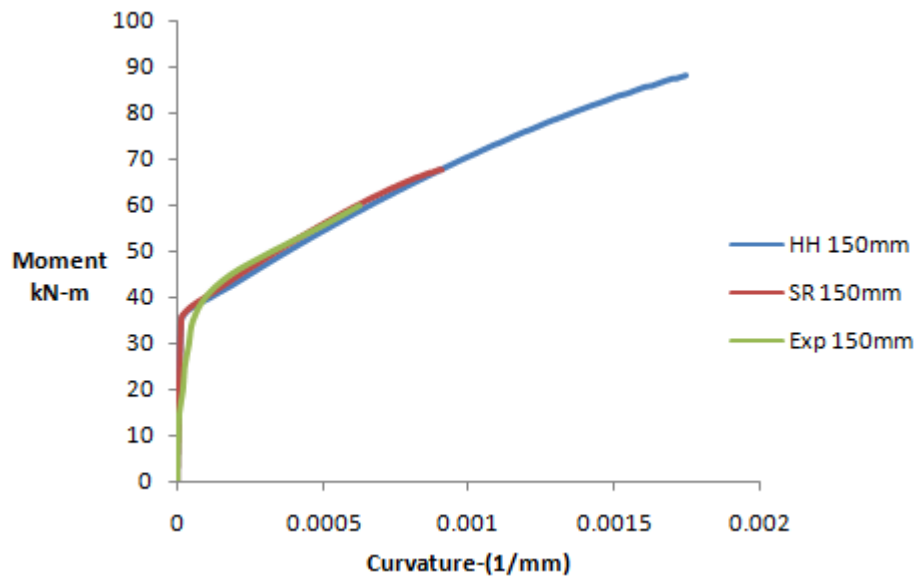
Graph 5.3 Comparison of experimental and analytical results for 2-legged 150mm c/c



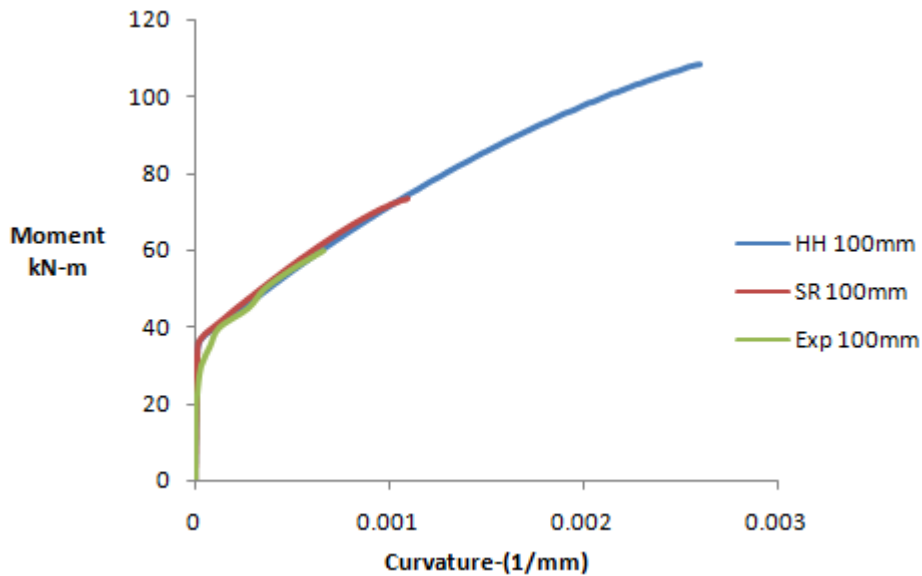
Graph 5.4 Comparison of experimental and analytical results for 2-legged 100mm c/c



Graph 5.5 Comparison of experimental and analytical results for 3-legged 250mm c/c



Graph 5.6 Comparison of experimental and analytical results for 3-legged 150mm c/c



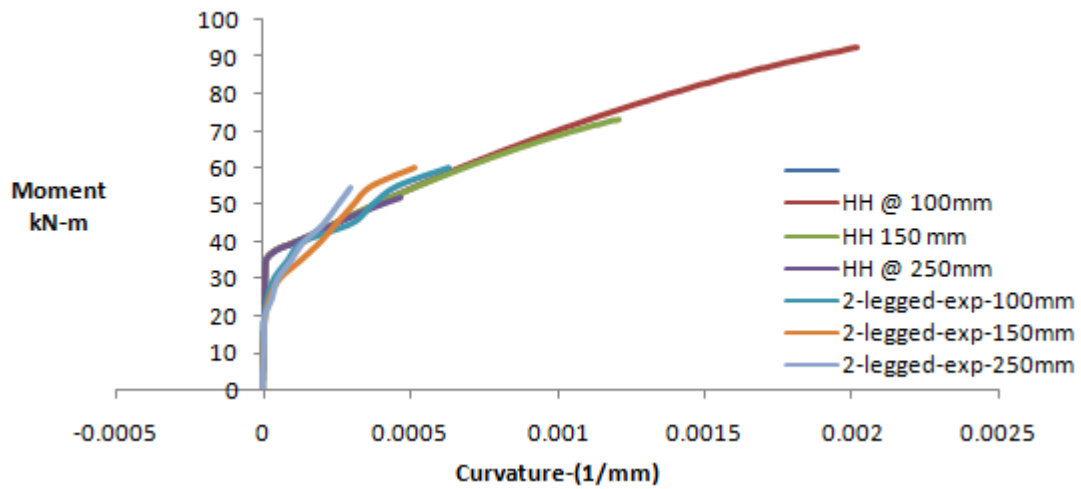
Graph 5.7 Comparison of experimental and analytical results for 3-legged 100mm c/c

The above graphs between Moment vs. Curvature is showing that curvature is increasing with decrease in spacing between the stirrups in the beam. At the same time there is a slight increase in curvature with increase in stirrup legs.

From the graph we can observe that there is a clear percentage increase in curvature for 2-legged beam is more than 3-legged beam.

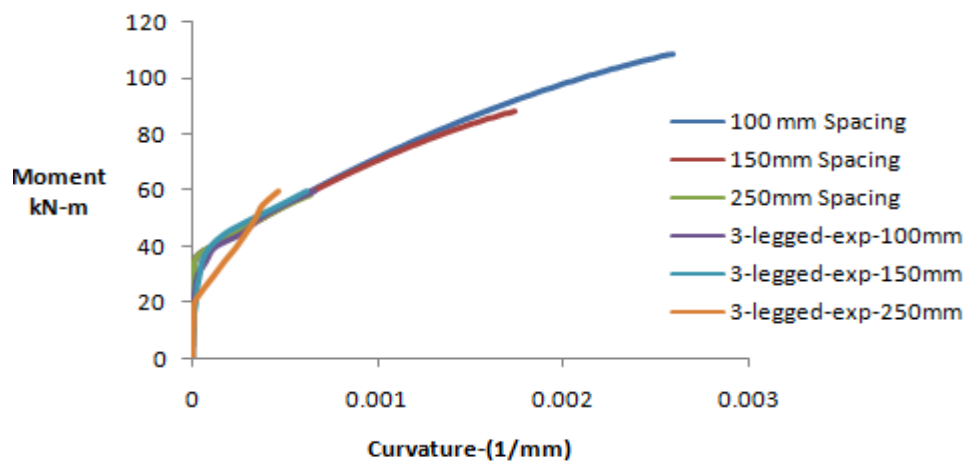
Percentage increase in curvature is maximum for 100mm and minimum for 250mm spacing.

Vs. Hong K N and Han S H Model (2005):



Graph 5.2 Comparison of Moment vs. Curvature (2-legged)

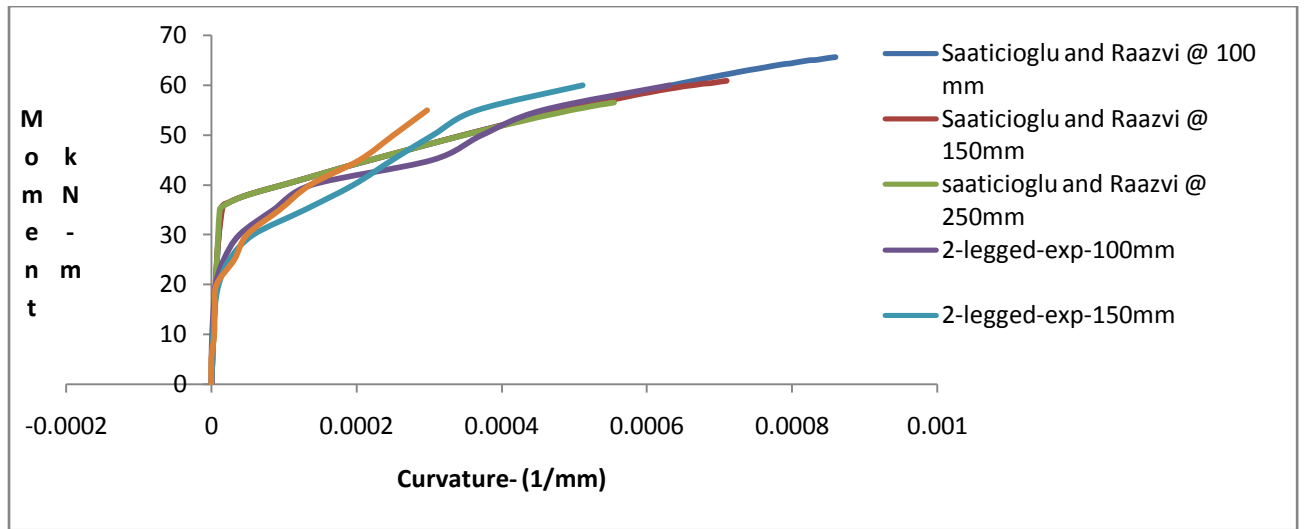
Analytical results are found to be 3-5 times more than the experimentally obtained values. In both cases curvature is increasing with decrease in spacing of stirrups.



Graph 5.3 Comparison of Moment vs. Curvature (3-legged)

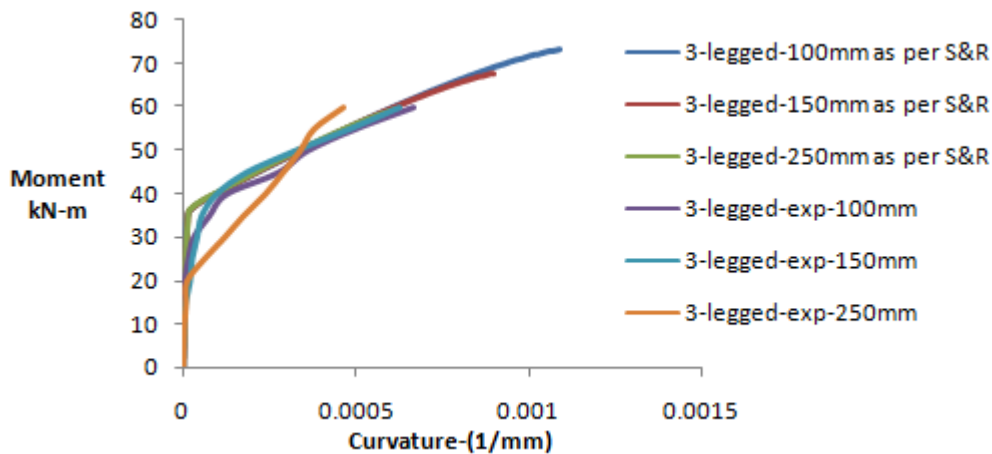
As per Hang K N Han S H (2005), there is not much increase in curvature as the stirrup legs are increasing.

Vs. Saatcioglu and Razvi Model (1992):



Graph 5.4 Comparison of Moment vs. Curvature (2-legged)

Experimental results are found to be nearer to the analytically obtained value. In this model also it is observe that there is an increment in curvature as the stirrup spacing decreases.



Graph 5.5 Comparison of Moment vs. Curvature (3-legged)

As per this model there is no considerable increase in curvature if the stirrup legs are increased. But experiment exhibited that there is a considerable increase in curvature as the legs are increasing.

Chapter-6

CONCLUSIVE REMARKS

6. CONCLUSIONS

- ✓ Stresses in concrete increase because of confinement and the corresponding strains are increases because of confinement.
- ✓ Hong K N and Han S H (2005) model is giving higher stresses and strains compared to the Saatcioglu and Razvi (1992) Model.
- ✓ Curvature ductility increases as the stirrup spacing decreases following **both** the confinement models.
- ✓ There is no significant increase in Curvature ductility if the stirrup's vertical legs increase.
- ✓ Experimental results are showing that the Curvature ductility increases as the stirrup spacing decreases.
- ✓ Hong K N and Han S H model is giving higher Curvature ductility values than the experimental findings.
- ✓ Saatcioglu and Razvi Model (1992) is found to be in good agreement with the experiment results.

Chapter-7

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